An Operational Model for Utilizing Water Resources of Varying Qualities in an Agricultural Enterprise

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The work considers the problem of determining an optimal operational plan in order to maximize the profits of an agricultural enterprise. A decision model is formulated considering a set of potential crops for planting on an available area of land, two sources of water supply of different qualities and limited capacities, and a production function for each crop using water quantity and quality as the input factors. A unique feature of the model concerns the nature of the production function, which measures the profit margin of a crop not only in terms of its yield, but also the quality of the products. The model was developed to assist local decision-makers in the Negev Desert (Israel) for the purpose of soliciting a preferred policy for water sources development. The developed model may be applicable for other regions with similar water shortage problems.

In the short run, the State of Israel is facing a gap between water supply and demand. The gap is primarily due to natural shortage and continuous deterioration of supplies. A general strategic approach dealing with historical water and land use in Israel is discussed by Feitelson (1996), who describes water policy issues from a macro point of view. In the present paper, micro water strategies are specified. More particularly, reducing the gap through the use of economic-geographic strategies should be considered to assist regional and state planners to optimize the net profit (revenue less expenses) of an agricultural enterprise. In a series of articles, Rabinowitz et al. (1988a, 1988b, 1992) and Oron et al. (1991) analyzed the economic value of an on-line, real-time, nonlinear mathematical programming model to assist local water allocation decisions for energy production and water supply for irrigation by an hydroelectric energy production system located at the Hazbani-Dan River, Israel. These works allow decision-makers located in the northern part of Israel to integrate local and national energy systems efficiently. The integration considers two main sources of energy supply: expensive national energy characterized by price variation; and, cheaper stochastic local energy supplied by the hydroelectric energy production system.

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A similar strategy combining local and national alternative water sources has been studied recently by Oron et al. (1996) and Brimberg et al. (1994; 1995) to assist local decision-makers in the Negev Desert in Israel for the purpose of soliciting a preferred policy for water sources development. The considered work differs from the previous ones, since it determines an optimal operational plan considering an agricultural enterprise consisting of a set of potential crops for planting in an available area of land and production function for each crop consuming water quantity and quality as the input factors. Therefore, the unique feature of the model concerns the nature of the production function.

The model presented here is tailored to the conditions in the Negev Desert of Israel, and therefore considers two main water supply sources: high quality (expensive water transported from the north of the country by the National Water Carrier (NWC); and, cheaper local saline groundwater (SW) of inferior quality. The developed model may be applicable for other regions, primarily arid zones, with similar water shortage problems.

The main focus of this article is to present a geographical economic issue dealing with an integrated local and national strategy aimed at supporting arid zones. Such a strategy may provide a balance between equity and efficiency across various regions or countries. To provide such a balance is critical in the Middle-East: water shortages are an obstacle to regional peace. The cost of implementing an optimization approach to improving the economic welfare of the whole region is negligible compared to the potential benefits. Thus, a formal model is formulated and solved in the next sections.

THE PROBLEM

Spiraling demand for high quality water, coupled with natural shortage and continuous deterioration of supplies, primarily in arid zones, has stimulated the search for alternative sources. The gap between supply and demand can be primarily closed by implementing two major strategic directions:

1. Importation of high quality water from external regions to the demand sites with limited developed water sources.

2. Gradual development of local non-conventional water sources. The step-by-step marginal sources development will be subject to local needs and future development prospects.

Three main non-conventional water sources can be identified in arid regions. The water from these sources can be used for diverse purposes; however, under some circumstances, it may require further treatment prior to application:

i) Runoff water (RW). Runoff is generated in regions with low soil infiltrability and sparse precipitation events. RW is mainly generated in arid regions such as the Negev Desert (Israel) during the winter season. Such regions are characterized by loess and other impermeable soils. Although a high quality is obtained, this water source has some drawbacks:

(a) RW generation is a stochastic phenomenon; hence, the reliability of water supply is relatively low;

(b) Under most circumstances, RW is generated relatively far away from the consumption sites, and hence, requires the use of transportation and temporary storage systems. This can be avoided to some extent through implementation of a microcatchment technology (Oron and Enthoven, 1987);

(c) Since RW is generated during a relatively short winter season, storage is required to satisfy high water demand during the summer period.

ii) Treated wastewater (TW). Treated wastewater (primarily domestic sewage) is a valuable water source. TW can be reused for a wide range of possible purposes, depending both on treatment level, and the precautions and control undertaken (Asano et al., 1992; Oron et al., 1993; Crook and Suramalli, 1995). The sewage treatment facilities are frequently located close to urban centers. Although the capital investment in sewage treatment plants is relatively high, the transportation of the effluent to the reuse sites is frequently less expensive (Lyon and Farrow, 1995). Combining sewage treatment with reuse possibilities as a solution for disposal with minimal environmental risks is also an attractive, complementary solution for water shortage problems (Oron, 1995).

iii) Saline Water (SW). Saline water can be obtained in the form of tail and drainage water in surface-irrigated agricultural fields or by pumping from deep aquifers (around 1,000 m deep) (Oron, 1993; Issar and Adar, 1992). The variable expenses associated with saline water pumpage from aquifers are high due to the depth. The salinity of the water is frequently above 4 dS/m [dS/m is a measure of the electrical conductivity (EC) of the SW]. Although the salinity makes this an inferior quality source, further treatment and primarily adequate application technology permit the use of SW with great economic advantages (Pasternak and DeMalach, 1987).

A decision model was formulated by Brimberg et al. (1994) for the development of marginal water sources at a regional level. This model was intended as an operational and investment decision aid for the medium-term planning and integration of water supply in the Negev Desert in southern Israel. The aim of the present work is to consider the local decision problem in detail. Given the water supplies of various qualities available at an individual demand center (an agricultural enterprise), the problem now is to formulate an operational plan to use this supply in the most profitable way.

THE MODEL

The objective of this section is to introduce a decision model that will assist local operational planners of a developing agricultural enterprise to establish an optimal trade-off between production output and the quality of water utilized. This type of problem is faced primarily in regions with limited high-quality water inventories. The model presented here is tailored to the conditions in the Negev Desert of Israel, and therefore considers two main sources of water supply, namely, high quality (expensive) water transported from the north of the country by the National Water Carrier (NWC), and cheaper local saline groundwater (SW) of inferior quality. Other sources such as TW and RW may be readily included in the model to generalize its applicability to different scenarios.

The System Variables

The input data required for the analysis is summarized by the following notation:

Ω – Set of n potential crops identified by the planners;

L – Total area (hectares) available to the enterprise for agricultural purposes;

V_w – Maximum supply (m^3/year) from the NWC available to the enterprise for agricultural use;

V_s – Maximum supply (m^3/year) of SW available from existing on-site pumping facilities;

S_w – Salinity (dS/m) of water supplied by the NWC;

S_s – Salinity (dS/m) of the SW from local sources;

γ_i – Upper bound on permissible salinity of water used for irrigation of crop i, i=1,...,n;

C_w – Cost (\$/m^3) of water obtained from the NWC;

C_s – Cost (\$/m^3) of pumping local SW and transportation to consumption sites.

The decision variables in the model are identified as follows:

A_i – Area (hectares) to be cultivated with crop i, i=1,..., n.

q_{wi} – Water allocated from the NWC supply for exclusive use by crop i, i= 1,...,n (m^3/hectare/year).

q_{si} – SW allocated for exclusive use by crop i, i=1,...,n (m^3/hectare/year).

Total water allocated to crop i is given by

$$q_i = q_{wi} + q_{si}\,, \qquad i=1,\ldots,n. \tag{1}$$

Since the values of q_{wi} and q_{si} specify the water mixture applied to crop i, a net water quality index Sm_i (for two water qualities) for this crop can be calculated as follows:

$$Sm_i = S_w\left(q_{wi}/q_i\right) + S_s\left(q_{si}/q_i\right), \qquad i=1,\ldots, n. \tag{2}$$

The above mathematical notation refers to two water sources only. Without loss of generality it follows to use an index i for the crop and to designate the various water sources by an index j. However, in order to emphasize the type of water source the alternative condensed notation was utilized. To complete the specification of the model, a production (or yield) function for each crop is needed. The production function is expressed in currency units, and takes into account the mixture of water quality applied. Different fixed charge costs were assumed for first order approximation for the water of the diverse sources. Usually the production is expressed by a non-linear function. However, for a first order approximation, and in a specific increasing range, up to reaching the moderate plateau range, a linear expression is valid. Substantial empirical data is available regarding crop yields as a function of the total quantity of applied water. However, the related studies generally involve the application of a single source of fixed water quality. Relatively limited empirical data exists on the effects that varying water quality has on output.

A new feature introduced by this model is the use of production functions that combine the yield and quality of a crop in an aggregate measure. The independent variables are the total water quantity and net mean quality supplied (q_i, S_{mi}). Thus, the production function measures the output in terms of the current or projected market value of the crop, recognizing that this value is affected not only by the yield of the crop, but also, in many cases, significantly by the quality (e.g., tomatoes, citrus fruits, melons, watermelons, pears, etc.).

Based on data available for a limited number of crops, the following form of the production function is deemed a suitable first order approximation:

$$f_i\left(q_{i0}, Sm_i\right) = a_{i1} + a_{i2}Sm_i\,, \qquad i=1,\ldots, n. \tag{3}$$

where a_{i1}, a_{i2} are parameters estimated from empirical results for a standard or reference quantity (q_{i0}) of water. Note that $a_{i1} > 0$, while a_{i2} is unrestricted in sign, since salinity may have a positive or negative effect on crop quality, depending on the type of crop i. Including the effect of a variable water supply, the following linearized form for the production function of crop i is proposed:

$$\begin{aligned} f_i\left(q_{i0}, Sm_i\right) &= \left[q_i/q_{i0}\right]\left(a_{i1} + a_{i2}Sm_i\right) \\ &= d_{i1}q_{wi} + d_{i2}q_{si} \end{aligned} \tag{4}$$

$$d_{i1} = \left(a_{i1} + a_{i2}S_w\right)/q_{i0}\,, \qquad i=1,\ldots, n, \tag{5}$$

$$d_{i2} = \left(a_{i1} + a_{i2}S_s\right)/q_{i0}\ , \qquad i=1,\dots, n. \tag{6}$$

The units of f_i (.) are in \$/hectare/year. The initial investment in each water source is neglected, although it is expected to be a linear additive term.

The decision model is formulated to maximize the net profit (revenues less expenses) as follows:

$$\text{maximize } Z\left(A, Q_w, Q_s\right) = \sum_{i=1}^{n} A_i f_i\left(q_i, Sm_i\right) - C_w\sum_{i=1}^{n} A_i q_{wi} - C_s\sum_{i=1}^{n} A_i q_{si}$$

$$= \sum_{i=1}^{n} A_i\left[\left(d_{i1} - C_w\right)q_{wi} + \left(d_{i2} - C_s\right)q_{si}\right] \tag{7}$$

subject to a set of constraints:

$$\sum_{i=1}^{n} A_i q_{wi} \le V_w \qquad \{\text{supply constraint, NWC}\} \tag{8}$$

$$\sum_{i=1}^{n} A_i q_{si} \le V_s \qquad \{\text{capacity constraint, SW}\} \tag{9}$$

$$\sum_{i=1}^{n} A_i \le L \qquad \{\text{available land}\} \tag{10}$$

$$Sm_i \le \gamma_i \qquad i=1,\dots,n \qquad \{\text{net water qualify requirements}\} \tag{11}$$

$$A_i, q_{wi}, q_{si} \ge 0, \quad \forall_i \qquad \{\text{nonnegativity constraints}\} \tag{12}$$

where $A = \left(A_1,\dots,A_n\right)$, $Q_w = \left(q_{w1},\dots,q_{wn}\right)$, and $Q_s = \left(q_{s1},\dots,q_{sn}\right)$ represent the set of decision variables. Recalling the formula for Sm_i, equation (11) can be rewritten as

$$\left(S_w - \gamma_{iI}\right)q_{wi} + \left(S_s - \gamma_i\right)q_{si} \le 0 \qquad i=1,\dots, n. \tag{13}$$

Note that the objective function Z has quadratic terms, as well as constraints (8) and (9). Meanwhile, (10) and (11) are linear constraints. It follows that the model takes the form of a nonlinear problem. Furthermore, since Z is neither concave nor convex, and the feasible region defined by the constraint set is nonconvex, the problem falls in the realm of global optimization (e.g., Bazaraa and Shetty, 1979; Horst and Tuy, 1991). In the next section, a simple heuristic algorithm for solving the model is proposed.

SOLUTION PROCEDURE

Given that the number of hectares A_i has been fixed for each crop i, the model reduces to a standard linear program (LPI) to solve for Q_w and Q_s. The values of the A_i must be chosen to satisfy constraint (10). Similarly, having fixed all the variables in Q_w and Q_s, the problem reduces once again to a linear program (LP2), this time to solve for A. Based on this observation, the following heuristic solution procedure is suggested.

A Local Improvement Algorithm

Step 1: Choose initial feasible values for A_i, $i=1,\ldots, n$;

Step 2: For the current set of values in A, solve LPI to obtain Q_w and Q_s;

Step 3: For the current set of values in Q_w and Q_s, solve LP2 to obtain A;

Step 4: Repeat steps 2 and 3 until the improvement in the objective function in two successive iterations is less than a specified value, or some other stopping criterion is satisfied.

The procedure outlined above will converge to a locally optimal solution. The quality of this final solution will depend on the initial estimates for the A_i, chosen by the analyst in Step 1. Thus, an interactive procedure can be used to improve the quality of the solution by repeating the local improvement algorithm for different sets of initial values of the A_i chosen by the decision-maker (analyst). In this way, the expertise of the decision-maker is utilized to construct several plausible starting solutions. The best candidate solution obtained from all the runs of the local improvement algorithm is chosen as the recommended solution.

An interesting feature of the decision model applies when a shortage of total water supplies exists (constraints (8) and (9)) such that we know not all the available land will be put to use. In other words, due to a scarcity of water, constraint (10) is non-binding. In this case, the problem reduces to a linear program by substituting

$$u_i = A_i q_{wi}, \qquad i=1,\ldots, n, \tag{14}$$

$$v_i = A_i q_{si}, \qquad i=1,\ldots, n. \tag{15}$$

The optimal solution may now be found by any of a number of standard linear programming software packages.

PRELIMINARY RESULTS

As noted previously, limited empirical data are available on the effect that water salinity has on the quality and yield of various crops. Table 1 gives parameter estimates for the production functions of three crops: tomatoes, cotton and corn.

Table 1: Parameter values for the production functions.

i	*Crop*	q_{io} m^3/*hectare*	a_{i1} \$/*hectare*	a_{i2} \$/*hectare*/(*dS/m*)
1	Tomatoes	7,000	6,752	+18.74
2	Cotton	8,500	4,836	−28.5
3	Corn	6,000	4,733	0.0

These estimates apply to typical soil and weather conditions encountered in the Negev Desert. Additional information required by the mathematical model relates to the cost and quality of alternate sources of water. Illustrative values are given by C_w = \$0.22/m³ and S_w= 1.1 dS/m for the NWC, and C_s = \$0.17/m³ and S_s= 4.4 dS/m for SW. It should, however, be noted that the parameter values for SW depend on the location of the source, and are highly variable from one local site to another. Limits on acceptable water quality for the crops being investigated also need to be specified: g_1 = 3.5 dS/m (tomatoes), g_2 = 8.0 dS/m (cotton), and g_3 = 2.5 dS/m (corn).

The parameters specified above may be used to draw some preliminary conclusions by examining the crop production functions (see equation (4)) and resulting profit margins. The revenue coefficients, d_{i1} and d_{i2}, for crop i are readily calculated from equations (5) and (6). The profit margins (\$/m³) for crop i are then evaluated as $(d_{i1} - C_w)$ and $(d_{i2} - C_s)$ for the NWC and SW, respectively. Table 2 summarizes the results for the three crops under consideration.

Based on the coefficient estimates in Table 2, the following general observations are made:

1. All three crops are profitable, with tomatoes providing a significantly higher profit margin than cotton or corn. Neglecting market externalities, our first reaction would naturally be to cultivate tomatoes only. This would be optimal if

Table 2: Revenue and profit coefficients.

		Revenue (\$/m³)		*Profit (\$/m³)*	
i	*Crop*	NWC (d_{i1})	SW (d_{i2})	NWC ($d_{i1}-C_w$)	SW ($d_{i2}-C_s$)
1	Tomatoes	0.9675	0.9764	0.7475	0.8064
2	Cotton	0.5653	0.5542	0.3453	0.3842
3	Corn	0.7888	0.7888	0.5688	0.6188

Table 3: An optimal scenario for limited water supply from the NWC.

Crop	*Water amount* m^3*/hectare*	*Water quality* *dS/m*	*Return for yield* *US \$/hectare*	*Yield quality*
Tomatoes	7,333	3.5	5,795	high
Cotton	8,000	4.4	3,075	medium
Corn		Not Cultivated		

there was an unlimited supply available from the NWC. However, the capacity constraints (8) and (9) on the alternate sources of supply cause an interdependence among the crops. That is, the water allocated to one crop reduces the amount left over for the others. Thus, by only cultivating a tomato crop? which requires a relatively high net quality of water, the limited supply from the NWC could be exhausted with only a fraction of the arable land being put to use. It would be more profitable in this case to plant fewer tomatoes in combination with crops requiring lower water qualities.

2. Comparison of tomatoes (crop 1) and corn (crop 3) shows that tomatoes provide a higher profit margin while requiring a lower net water quality. We can say that crop 1 dominates crop 3, and hence, it makes no sense to cultivate crop 3. The concept of dominance is useful in this context, since it allows us to eliminate through pairwise comparisons crops which cannot be included in an optimal solution, thereby simplifying the analysis.

3. For the small problem at hand, an optimal solution may be constructed manually without recourse to a solution algorithm such as the one described above. Since the profitability of tomatoes increases up to a specific salinity of the water supply, the mixture of NWC and SW which gives the maximum allowable salt content (γ_1) will be applied. This fixes the relative amounts of NWC and SW applied to the tomato crop. The entire supply from the NWC will be allocated to the tomato crop. Some or all of the remaining hectares of land may then be planted with cotton, which will be irrigated by the left-over SW capacity.

4. A potential scenario for limited water supply from the NWC can be described. Since under these conditions saline water will be the main source, only two crops will be cultivated (Table 3).

CONCLUSIONS

A decision model is formulated to assist local planners in choosing the most profitable mix of crops in an agricultural enterprise. The model is applicable to an arid region such as the Negev Desert in Israel, where alternate sources of water of varying qualities must be used due to scarcity of supply from a main high-quality source.

The various water supplies must be blended and applied in a judicious fashion, and the available land must be partitioned among the chosen crops, in order to make optimal use of the limited resources.

A unique feature of the decision model is the use of a production or yield function for each crop, which estimates the quantity and quality of the crop in terms of a net market value as a function of the volume and blend of water applied. Thus, the objective of the model is to maximize anticipated profits subject to resource constraints on land and alternate supplies of water, as well as a net water quality requirement on each type of crop. Another application of the model may be as a strategic planning tool to evaluate the operating profits of various proposals to expand local sources of water supply.

A simple linearized form of the production function is assumed here, from which the marginal profitability of a given crop may be readily estimated to provide much-needed information to management. Future research should be directed to the empirical validation of the production function for a wide range of crops, and to possible extensions of the decision model.

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