On the Measure of Geographic Segregation

Richard L. Morrill University of Washington, U.S.A.*

Although measures of segregation are widely discussed, this paper presents a critical review of several available indices, with a special focus on their spatial attributes. These indices are applied to a hypothetical case, as well as to the case of Indianapolis. The results suggest that if interest is a-spatial, then traditional indices of dissimilarity are satisfactory structural measures of segregation over a data unit. However, if a meaningful measure of territorial segregation is desired, then an interaction-based measure appears to be the most consistent and desirable approach.

Segregation is one of the most fundamental of human geographic processes, and therefore its consistent and appropriate measurement is also important. In this brief paper, I evaluate a few of the more common measures, and propose a "geographic" variant of the most popular measure, D, the "index of dissimilarity." The knowledgeable reader will be aware not only of a fairly large and old literature on the question of the measurement of segregation, but of a recent, rather thorough review by Massey and Denton (1988). Nevertheless, I embark on yet another commentary, and suggest yet another variant, for two reasons: first, there is still rather universal use of the "index of dissimilarity", D, despite well-recognized problems, especially for territorial (geographic) research; second, segregation is such a basic variable in social science—especially in geography and sociology—that it deserves our best thinking and periodic rethinking.

What does segregation mean and what is it we want it to measure? Segregation refers to the spatial separation of groups, resulting from certain physical or social processes. For example, populations of plant and animal species settle in, and compete for, and come to dominate and defend, particular territories in response to conditions of the physical environment, competitive characteristics of the species, etc. Humans similarly become segregated on many bases—language, religion, allegiance, class, lifestyle, and especially ethnicity and race, via processes perhaps not so different from plants and animals, but involving even more variables.

Competition for space, and the varying capacity to compete, and varying motivations for competing are just as basic. Thus racial and ethnic segregation, for which

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^{*} Department of Geography, University of Washington, Seattle, WA, USA.

measures are commonly reported, is a consequence of a social process characterized by strong group preferences, perceived incompatibilities between groups, and variable power to compete for space. Segregation is not accidental, but a consequence of purposeful behavior. It is at once a structural and a spatial concept (Blau, 1977; Boal, 1987; Jackson, 1987). The motivations for segregation are structural—that is, a desire to minimize interaction with certain other kinds of people. The most effective tool or manifestation is territorial separation.

A quality index of segregation may measure, first, the degree to which a group is concentrated in particular territories, what Massey (1978) calls "evenness"; or, second, the degree to which a group dominates or shares particular territories (what he calls "exposure"); or, third, the probability of, or degree of, contact between members of different groups as a result of their mutual segregation (what he calls "clustering"). The first two are structural dimensions of territorial domination or group concentration. The third, however, also captures the spatial component of segregation, and is, therefore, a more meaningful concept to a geographer because it gets to the heart of the process that creates segregation—the intent to reduce contact between presumably incompatible social groups.

Massey and Denton (1989), recognizing the limitations of the traditional D measure, suggest that there are in fact five dimensions, adding two others: "centralization" with respect to the central business district, and "concentration." These are discussed below.

For example, the "index of dissimilarity," D, which is an aggregate measure of the concentration or "overrepresentation" of one group in areas which it dominates, is of the first type; the "exposure" and "isolation" indices, which measure the extent to which members of a minority share territory with members of the majority, are of the second type; the simple location quotient, which compares a territory's proportion of the group in question to the area's proportion of the total population, is similar; the 'third kind of measure requires the explicit treatment of the relative location of groups. "Distance-based" measures that modify the index of dissimilarity are an attempt at the third type, but only White's (1983) "proximity" measure, based on spatial interaction, directly concerns the probability of contact as a function of distance as a basis for measuring segregation. My "boundary modified" variant of D is, like the "distance based" modifications of D, an attempt to combine structural and spatial dimensions into one measure. Although useful, Massey and Denton may be correct that segregation is inherently multi-dimensional, and that we should routinely report multiple measures.

COMPARING THE MEASURES

The following measures will be briefly reviewed:

Structural (non-spatial) indices

- 1) The index of dissimilarity
- 2) Isolation index (mean weighted percent black)
- 3) Exposure index

(1)

Spatial Indices

- 4) Distance-based index of dissimilarity
- 5) Spatial autocorrelation
- 6) Spatial interaction ("proximity")
- 7) Centralization index
- 8) Concentration index
- 9) Boundary modified index of dissimilarity

There should be no need to repeat the details of the large body of earlier critique and literature. The reader is referred to Duncan and Duncan (1955), Darden (1987), Taeuber and Taeuber (1965), Lieberson (1981), Winship (1977, 1978), Cortese et al. (1976), White (1983), Morgan (1982, 1983), Jakubs (1981), Van Valey and Roof (1976), Massey (1978), Lieberson and Carter (1982), Guest and Weed (1976), Spriggs (1984), Massey and Denton (1985, 1988, 1989), Farley (1984), Massey and Mullan (1984), Stearns and Logan (1986), Langlois (1985), Jackson (1987), Galster (1988a), and Clark (1986). It will be helpful to compare the various measures, first via some simple, hypothetical patterns, which test the "flexibility" and consistency of the measures, and then through a real example—the city of Indianapolis. The hypothetical patterns include three at a general black proportion of 25 percent, with sub-territories either white or black, but of differing spatial arrangement; and two with the same average proportion black, but with some of the basic units at 50 percent and some at 100 percent black/white, and again in varying spatial patterns (Figure 1).

STRUCTURAL (NON-SPATIAL) INDICES

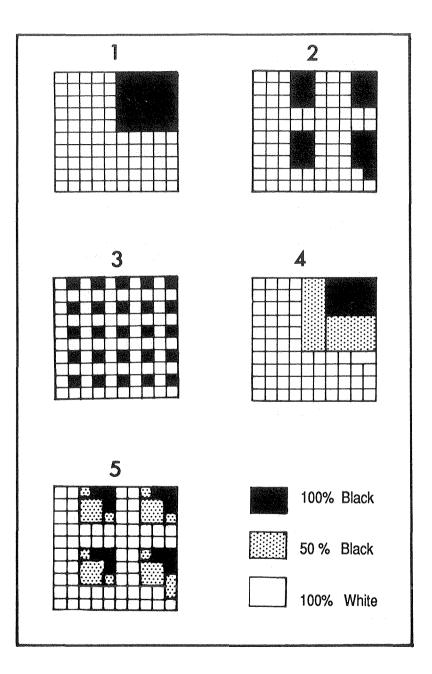
(1) Index of dissimilarity, D

$$\mathbf{D} = \frac{1}{2} \sum \left| \frac{\mathbf{b}}{\mathbf{B}} - \frac{\mathbf{w}}{\mathbf{W}} \right|$$

where b is the black population in any area, w is the white population in any area, B is the total black population, and W the total white population. That is, D is the sum of the positive or negative differences in the proportions of the total black population and the total white population residing in each area. It is interpreted as the proportion of the minority population that would have to move for all areas to have the same average proportion black (see Duncan and Duncan, 1955; Cortese et al., 1976; Massey, 1978; Jakubs, 1981).

The main problem with this as a measure of a spatial phenomenon is that it is a structural measure only—of concentration or evenness across areas, not of segregation in a geographic sense. By definition, for any pattern in which subareas are either black or white, regardless of size or spatial placement, the index of dissimilarity must be 100; that is, it is completely insensitive to whether there are single or multiple clusters of minorities. Therefore, for patterns 1 to 3, with one, four, and 25 clusters of blacks,

Figure 1: Sample patterns.



D is invariant at 100, and for patterns 4 and 5, with one or four concentrations, D is always 83. Yet intuitively, the probability of interaction is assuredly greater in pattern 3 than in 2 than in 1, and in 5 than in pattern 4.

A minor intuitive problem in D is that only the minority population moves—in the example, patterns 1 to 3, totally abandoning the one-fourth of the area that was black. This has no effect on the measure for comparison across cities, but alternatively one can separate the proportion of blacks and whites "moving" if equal numbers moved, so that all areas not only have the same average proportion, but also the same total population as before—just as in an ideal busing-for-desegregation plan. The proportion of blacks and whites will be in the ratio of W to B (that is if W/T = 0.75, as in these patterns, then 75 percent of blacks and 25 percent of whites will move, and the sum of the proportions will exactly equal D. Thus in patterns 1, 2, and 3, if only blacks move, all 250 do so, and no whites (the usual, but unrealistic implication); if blacks and whites move, then 0.75 of 250, or 187.5 blacks and 0.25 of 750, or 187.5 whites, move (375 altogether); and in patterns 4 and 5, 62.5 percent of blacks and 20.8 percent of whites would have to move. In Indianapolis, instead of 74 percent of blacks (115,000) having to move, 59 percent of blacks (91,500) and 15 percent of whites (91,500) would move.

Another way of seeing how serious the matter of the pattern of units is can be shown by conducting a "sensitivity" test of the grid system. In patterns 1 to 5, consider if the grid were moved one-half a cell-width to the right, over the distribution of the minority. This would have the following effect on D:

pattern 1: 90 instead of 100 pattern 2: 74 instead of 100 pattern 3: 50 instead of 100 pattern 4: 60 instead of 83 pattern 5: 44 instead of 83

The one big cluster is only slightly affected, while the greater the number of clusters, the greater the decline in D. This illustrates that somehow spatial pattern must be taken into account.

(2) Isolation index or mean weighted percent black

This is the Shevsky index as revived by Lieberson (1981) (see also Farley, 1984; Massey and Denton, 1985; 1988).

$${}_{b}p^{*}{}_{b} = \sum b/B(b/t)$$
⁽²⁾

where t is the total population in any area, b as before. Then:

$$_{b}p_{w}^{*} = 1 - _{b}p_{b}^{*}$$
 and $_{w}p_{b}^{*} = _{b}p_{w}^{*}(B/W)$, etc. (3)

 ${}_{b}p^{*}{}_{b}$ is simply the "percent black," experienced by the average black person, which is intuitively somewhat simpler to understand than the index of dissimilarity. As an

"asymmetrical" index, it also recognizes that "segregation" differs for the majority; e.g., that whites are far more segregated from blacks than the reverse, since blacks are usually a minority.

Unfortunately, this measure is, similarly to the index of dissimilarity, structural only and is just as insensitive to spatial arrangement. Thus in patterns 1 to 3, the index remains 100, and ${}_{b}p^{*}{}_{w}$ and ${}_{w}p^{*}{}_{b}$ (intergroup contact) are both 0. In patterns 4 and 5, ${}_{b}p^{*}{}_{b} = 74$ (120 at 100%, 130 at 50%), then ${}_{b}p^{*}{}_{w} = 26$ and ${}_{w}p^{*}{}_{b} = 8.7$, and ${}_{w}p^{*}{}_{w}$ is 91.3. In Indianapolis, ${}_{b}p^{*}{}_{b}$ is 78.5.

(3) Exposure Index

The exposure index is expressed by:

$$EI = \frac{1 - \sum b(w/t)}{B(W/T)}$$
(4)

This index is not really very different from the isolation index (see Winship, 1977; Massey and Denton, 1988). By examination of the formula, it is seen that it adjusts for composition differences through the ratio term (W/T), and is necessarily lower than ${}_{b}p^{*}{}_{b}$, except where subareas are either all black or all white, where again the index will necessarily be 100. Thus in patterns 4 and 5, the exposure index drops to 65.3 (in Indianapolis, it is 58.3). This index offers no particular intuitive advantage over D or ${}_{b}p^{*}{}_{b}$, although it appears to be closer to the results than an interaction-based index (see below).

SPATIAL INDICES

(4) Distance-based modification of D (DBI)

Jakubs (1981) well understood the limitations of D, and attempted to overcome its spatial insensitivity by asking the question: how much total travel would people be required to undertake for all areas to have the same average proportion black? The travel could be viewed either as by blacks alone (see Morgan, 1982; 1983) or as by both groups:

$$DBI = D^* / D^*_{max} \quad \text{where } D^* = Min \sum bd$$
(5)

where D^* is the distance traveled for the observed pattern and D^*_{max} is the distance traveled from a maximally segregated pattern with the same proportions and area.

The measure requires calculating the distance that would be traveled from the most segregated possible pattern, which is in itself not easy to find; both D^* and D^*_{max} require use of the transportation problem of linear programming, whose set-up is rather tedious. The index is an ingenious idea, but in the example patterns, the measure does not behave very well. Since, by construction, pattern 1 is the most segregated possible (maintaining contiguity), DBI would also be 100 (or 75), if both blacks and whites

Measure of Geographic Segregation 31

moved; whereas intuitively, there should be at least a little exposure across the common B/W boundary. Conversely, in pattern 2 DBI drops to 50 and in pattern 3 to 20, which again intuitively seems to have the opposite bias—less separateness than probably exists, given the patterns. Similarly, DBI for pattern 4 is like D for pattern 4, but half D for pattern 5.

(5) Spatial autocorrelation

One plausible candidate that is computationally simpler, and which explicitly treats the relations between areas and across boundaries, is to measure the spatial autocorrelation. This is basically the simple correlation coefficient r, but the data are the proportions black across adjacent boundaries. Thus, if proportions are similarly high or low, a high positive correlation implies high segregation. The correlation measure is:

 $r_{Z_i Z_j}$ (6)

where z_i and z_i are proportions black in adjacent areas.

The correlation measure has a certain ambiguity about it, however. For pattern 1, r is 0.92, and for pattern 2, 0.73, but for pattern 3 it is -0.58. That makes sense, because the checkerboard pattern is more evenly interspersed than a random placement (r = 0) would be; but this is not a satisfactory measure of segregation, because even pattern 3 exhibits spatial separation, even if at quite a local level. For patterns 4 and 5, the r's are 0.94 and 0.74, and seem reasonable, as does the 0.71 for Indianapolis. Yet if all areas were "integrated" with the very same proportion of 25 percent, the autocorrelation would be 1.0.

(5a) Trend surface measure of segregation.

Spriggs (1984) proposes a simple trend surface calculation using the percent black as the z value, and the x and y coordinates. As he shows in the case of Norfolk, VA, this calculation will be sensitive to whether there is one big minority area or multiple clusters; a high R^2 or regularity of surface indicating a high degree of segregation. This idea is also clever, but is very dependent on the complexity of the surface calculated.

(6) Spatial interaction / proximity measures

The concept of spatial interaction as embedded in the familiar gravity model and its variants may be explicitly interpreted as measuring the propensity or probability of contact between populations across space; exactly what we are really looking for in a segregation measure. Intuitively, what is needed is a measure of the degree to which a group's contacts are intra-group, as black to black, or inter-group (as black to white), *because* of the spatial arrangement. White (1983) proposed a "proximity" measure, based on spatial interaction principles, and this is adopted by Massey and Denton (1989) as the preferred spatial or "clustering" measure. As with the isolation or exposure index, White was interested in black-black, black-white and white-white exposure, but he suggests using the gravity model to calculate I_{tt} , I_{ww} , I_{bb} , and I_{ww} , where each

is a kind of "per capita" measure of interaction, since B², for example, is by definition maximum intra-group interaction, at "0" distance. E.g.,

$$I_{bb} = \frac{\sum b_{i}b_{j}/f(d_{ij})}{B^{2}}$$
(7)

and the index is:

$$p = \frac{B I_{bb} + W I_{ww}}{T I_{tt}}$$
(8)

where a p>1 is more segregated than random. If blacks are closer to other blacks and whites to other whites than the groups are to each other, then the sum of their in-group interactions will be greater than "color-blind" interaction over the total population. I find it preferable to use White's building blocks, I_{bb} , I_{bw} , I_{ww} , to form a much simpler segregation measure: for the minority,

$$Seg = I_{bb}/I_{bb} + I_{bw}$$
(9)

that is, the proportion of all black interaction that is within the group.

Use of this spatial interaction approach raises several technical issues. These are not trivial questions; the choices greatly affect subsequent results:

- the distance function: traditional gravity model d^{-b}, or negative exponential, e^{-bd}. The more familiar D^{-b} is quite satisfactory, although Massey and Denton (1989) use a standard negative exponential, e^{-d}. The difference here is not nearly as consequential as the exponents chosen. The problem of self-distance, or distance within each unit, can be a problem, especially with the gravity version.
- 2) the appropriate exponents or friction of distance, and whether these should differ within and between groups.

Presumably these exponents can be determined by actual studies of social interaction; for example: marriage distance data; exponents between groups might be as much as double that within groups, the propensity to interact across races being less likely even on the same floor of the same buildings—a structural influence on a spatial measure!

3) whether to use all information (each subunit to all other subunits) or a restricted zone around each area.

If x and y coordinates of areas are used, there is no computational or data problem, but if real distances or times are used, restriction to a smaller zone is perhaps justifiable, because the friction of distance is high enough to discount distant subunits.

For the example patterns and Indianapolis, the simple gravity model, with D^{-2} , was used with no distinction between intra- and inter-group exponents. All the results are shown in Table 1. Self-distance was set at 0.5 (the distance to a neighboring cell is 1). The fact of the common border, with two rows of cells interacting on either side, is sufficient to reduce the internal I_{bb} to 94 percent of the total $I_{bb} + I_{ww}$, even in the most segregated pattern 1. Pattern 3, the checkerboard, yields a ratio of in-group to total

Measure of Geographic Segregation 33

interaction of 60, by far the most intuitively reasonable measure for that pattern. The spatial interaction measure easily distinguishes between patterns 1, 2, and 3, as the other measures cannot, and similarly, between 4 and 5. For Indianapolis, the proportion of black interaction within the group, by location, is 55.4 percent (and for whites, 88 percent). These results are fairly similar to the exposure index (58 percent) and the proportion of blacks who have to move (59 percent) if *both* groups moved, according to D. In general, values were lower than for D and for the p* measures, as Massey and Denton (1989) also found. However, if cross-group propensities to interact were half in-group propensities (i.e., a different friction of distance) then the Indianapolis value would be 71, instead of 55.

Index Pa	ttern: 1	2	3	4	5	Indian- apolis
D	100	100	100	83	83	74
D* (both move)	75	75	75	62	62	59
ьр [*] ь	100	100	100	74	74	67
Exposure index	100	100	100	65	6	58
DBI	100	50	20	83	42	?
r _{zz} (autocorr)	0.92	0.73	0.58	0.94	0.74	0.71
Spatial interaction	94	83	60	66	54	55
D (boundary adj)	94	83	48	76	66	62

Table 1: Values of segregation indices.

Another advantage of this kind of index, which explicitly includes actual distance, is that the measure to a degree might be able to handle change in scale or aggregation, because as the scale goes up, distance increases and interaction falls. What the index tells us is the expected proportion, based on location and numbers, of interaction within and between groups. Thus, even if one population is racially homogeneous in small units, the greater the extent to which they border units of different races, the greater the opportunity for interaction—the principle behind busing desegregation.

The behavior of this simple ratio version of the spatial interaction index, at the extremes, is from 0 to 100—if there were no in-group contact across households, clearly a hypothetical rarity; and 100 if blacks occupied a separate territory, as with a reservation or with perfect apartheid. Normally, the expected minimum value for no segregation or random location of groups would simply be the proportion black; for patterns 1 to 5, it would be 25; for Indianapolis, 20.2; in a city which was 70 percent black, it would be 70. Should such a measure then be independent of the city-wide pro-

portion black? No, because high proportions minority are themselves prima facie evidence of segregation.

(7) Centralization

Massey and Denton (1989) consider centralization of a minority group with respect to the central business district as a dimension of segregation. While centralization of any group is a meaningful measure of relative position in the metropolis, it is not logically a measure of segregation (that is, the purposeful separation of groups). For example, in many southern cities, the measure is very low, simply because segregation is by sector, or because blacks are also in low-income suburbs or outskirts, with low levels of service. (see also Glaster, 1988b).

(8) Concentration

This measure, adapted from a regional science measure, is an averaged form of the location quotient, which compares, for any area, its share of a particular group to its share of the total population. As such, it is structural, and essentially like the exposure index above. In the form suggested by Massey and Denton (1989), it is spatial but not interactional; it measures the relative density or area concentration. Again this may be an interesting comparative statistic, but is not a measure of segregation, since, like centralization, a highly segregated minority group could reside at the same or even lower densities than the majority.

(9) Modifying the index of dissimilarity by analysis of boundaries

A possible way to combine the structural dimension of segregation embedded in the D measure and the spatial dimension of segregation is to adjust D via an analysis of the pattern of differences in proportions minority across all adjacent boundaries. This is the same idea as that behind use of spatial autocorrelation, except by simply calculating the mean difference in proportions black, and subtracting this mean difference value from D, it represents the structural index of segregation. The logic of this is as follows: if a very high proportion of the common boundaries with other tracts show a similarly high or low percent minority, as in pattern 1, then D is meaningful as it is, because there are limited opportunities to interact across space; but if a high proportion of the common boundaries show a big minority-majority difference, as in pattern 3, a high degree of opportunity to interact across space is present. D can be modified to take those differences into account.

$$D(adj) = D - \frac{\sum |z_i - z_j|}{E}$$
 (10)

where z_i and z_j are proportions black in adjacent cells and E is the number of edges. For pattern 1, there are ten 100% differences and 170 0% differences, for a mean difference of 5.5 percent. For pattern 2, the mean difference rises to 17 percent and for pattern 3, 52 percent. For pattern 4, the mean difference is again 5.5 percent and for pattern 5, 17 percent. For Indianapolis, the mean difference is 11.5 percent. At the extreme, if the

Measure of Geographic Segregation 35

black concentration were physically separate from the white areas, the mean difference would be 0 and D would be unaltered. There is a simple and direct relation of pattern the number and size of clusters—and the degree of adjustment. As there are more boundaries with a major difference in minority proportions, the probability of interaction increases. Thus, the mean difference is a kind of substitute for a spatial interaction measure; its use to modify D may be a way to combine structural and spatial dimensions of segregation. Its calculation, however, is probably no simpler than a spatial interaction measure, and there is the issue of how much difference varying lengths of edges (boundaries) might make. More experimentation across cities will be needed to evaluate its consistency and utility.

CONCLUSION

If interest is not geographic or territorial, then D or the p^* exposure indices remain satisfactory structural measures of racial dissimilarity over data units, but if a meaningful measure of territorial segregation is desired, then an interaction-based measure appears to be the most consistent and desirable approach. The alternative, to modify D by subtracting the mean difference in minority proportions across common boundaries, may be useful. If the phenomenon under study is territorial, and the process is one of competition for space, then it is illogical to pretend that information on the data units themselves will be sufficient; focus must also be on the territorial relationships between units. Segregation is multi-dimensional, but probably of two or three dimensions, rather than five. It is spatial, as best indicated by a spatial interaction-based measure, and it is structural, as revealed by either the sense of territorial (un)evenness by D, or by relative sharing of territory by the p^* isolation or exposure indices.

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