Household and Business Firm Densities in the Danish Urban Pattern

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The purpose of this paper is to expand the discussion of urban population density in two ways. The first is to include the business firm density in order to get an impression of the interrelationship between firm and population densities. The second is to estimate a comprehensive model including a description of the densities in 22 towns, making the coefficients of the individual town equations a function of population size, income and employment pattern, using the expansion method.

In the theoretical framework developed for discussing urban population density, the standard assumption is that all urban land is used for residential purposes, with working places and shops placed at a point in the center (see, e.g., Alonso, 1964; Thrall, 1980; Wheaton, 1974; 1979). Spatial equilibrium is attained when rising traveling costs to the center are compensated by declining land rent in such a way that household welfare is spatially invariant (Alonso, 1964; Casetti, 1971). This theory results in a declining density function from the center to the periphery, which has been estimated in some variant of a negative exponential function (see, e.g., Mills and Tan, 1980; Alperovich, 1983; and the survey articles of Anselin and Can, 1986; and McDonald, 1989). The shortcoming of these models is that they do not recognize the population density crater.

The use of land for more than one purpose was already introduced by Von Thünen (1826) and also mentioned by Alonso (1964). The subject is later touched on by a number of authors (e.g., Thral, 1980; Wheaton, 1979) who place emphasis on the rapidly rising number of problems resulting from the increasingly sophisticated nature of the models. Furthermore, the classical description of population density seems to give an unbalanced picture of city development, referring as it does to "loss of population" or "gain of population." Parr (1985a) briefly mentions the distinction between "daytime" population and "residential" population, and Parr (undated) mentions the city-center as "used increasingly for non-residential purposes."

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Without directly introducing other applications for urban land than for residential purposes, "other" purposes have been introduced indirectly in empirical investigations by introducing mathematical formulations of the density curve, which are able to catch a city center density crater if it exists. The density function in this paper adheres to the discussions of Amson (1972), Zielinski (1978) and Parr (1985a). A model based on BoxCox transformed data was tried by Kau and Lee (1976) in order to find a model which encompasses more of the above-mentioned functions in one equation. A similar procedure was tried in this project, but the results were not convincing, and were finally abandoned.

In this paper it will be shown that population growth leads to greater concentration of business firms in city centers, which again leads to the opening of the population density crater. Although the exact interrelationship between firm density and population density via the price mechanism (see Hansen and Kristensen, 1989) cannot be established due to major individual differences between the cities as stated below, this paper indicates a simple and plausible empirically-based relationship which can compete with more theoretical explanations (see, e.g., Beckmann, 1969; Amson, 1972).

Discussing the question of city population density, the literature has been concentrated on the description of individual towns rather than metropolitan and adjacent areas (see, e.g., Blumenfeld, 1954; Parr, 1985b). More comprehensive models, discussing cities in an intra-urban, as well as an inter-urban, context, as seen by Parr and Jones (1983) and Alperovich (1983), are rare.

Using the expansion method as proposed by Casetti (1972; 1973), the density profile for 22 Danish towns is included in one model, which can be seen as a cross-section parallel to the time-series analysis of city growth and population density made by Krakover (1983; 1985), and parallel to the cross-section study of population density made by Alperovich (1983).

DATA

As the basis for this investigation, a representative sample of telephone subscribers was selected from the telephone book. The distance from the addresses to the city center was measured and the number of subscribers within 250 meter rings was noted. The area of the rings was calculated and the telephone density was determined for each ring. In Denmark, the density of telephones can be taken as quite a good approximation of the density of households. The difference between household density and population density is mainly a question of number of children per family. Related research seems to indicate that the number of children rise with increasing distance from the center. This problem will, however, be neglected below. The data was grouped as private telephones and business telephones.

For coastal towns the ring area was calculated by drawing radians through the city center (town hall) as close to the coast as possible. The implication of this method is, however, that a small part of the town will be outside the investigated area. In recalculating sample to population, it is assumed that the actual distribution of population inside and outside the considered area will be equal to the number of observations within and outside the area. The formula used for calculating the densities of a particular ring was:

$$D = N*spi*POP/(A*OBS*V) \quad (1)$$

D	–	density
N	–	number of observations in the ring considered
spi	–	share of observations in the ring considered which are within the area considered.
POP	–	town population
A	–	area of the ring
OBS	–	total number of observations for the town considered
V	–	measure for the angle for the town share included. $360^{\circ} = 1$

If the population density is D and the distance to the city center is t, a number of possible, functional relationships can be set up. Parr (1985a) gives an elaborate discussion of the required characteristics of candidate functions for description of population density in a metropolitan-area-based region. In this project, a spectrum of functional relationships was tried in order to let the coefficients and the determination coefficient reveal the form of the density curve. The basic form of the investigated relationships which seemed most powerful was

$$D = a_0 + a_1 lt + a_2(lt)^2 + a_3(lt)^3 \quad (2)$$

where D has two versions:

DH	–	density of households
DF	–	density of firms
t	–	distance to center
lt	–	log(t)
a_j	–	coefficient s

In comparing different towns, data for population size, occupational distribution of daytime population, and income are included. Population is defined as the Statistical Yearbooks data for population in the individual urban areas, while income is the average income of the municipality in which the town is situated. Occupational distribution of daytime population is used as defined by Statistiske Efterretninger based on municipality data using 11 categories.

THE ESTIMATED DENSITY CURVES FOR TWO SELECTED TOWNS

Density for households and business firms

The density curve for households in Danish towns seems to divide into two groups: one showing the classical pattern of a density crater; and the other, where the density crater has not opened to a degree which can be caught by the estimations. The larger a town, the more likely it is that the density crater has opened.

Copenhagen (1,170,116 inhabitants, here defined as including Copenhagen, Frederiksberg and Copenhagen County) and Viborg (29,409 inhabitants), according to 1986 figures, seem to represent the two types of towns.

Copenhagen (with a density crater)

									$\bar{R}^2$	OBS
DH_K	=	− 548.82	+	219.001t	−	$27.31(1t)^2$	+	$1.09(1t)^3$		
		(−5.72)		(5.63)		(−5.32)		(4.93)	.70	60
DF_K	=	1149.36	−	398.501t	+	$45.91(1t)^2$	−	$1.75(1t)^3$		
		(13.13)		(−11.22)		(9.80)		(−8.71)	.92	60

Viborg (without a density crater)

									$\bar{R}^2$	OBS
DH_{Vi}	=	132.04	−	56.961t	+	$8.34(1t)^2$	−	$.4097(1t)^3$		
		(3.20)		(−2.94)		(2.81)		(−2.74)	.90	16
DF_{Vi}	=	127.42	−	49.061t	+	$6.31(1t)^2$		$.2715(1t)^3$		
		(6.37)	−	(−5.23)		(4.39)		(−3.75)	.98	16

The four estimated relationships are shown graphically in Figures 1 and 2. Figure 1 leaves the impression that when towns achieve a certain size the population is squeezed out by business firms. This effect will be investigated further in the next sections.

Figure 1: The densities for households and firms estimated for Copenhagen. (Each interval indicates 250 m; axis begins 125 m from the center.)

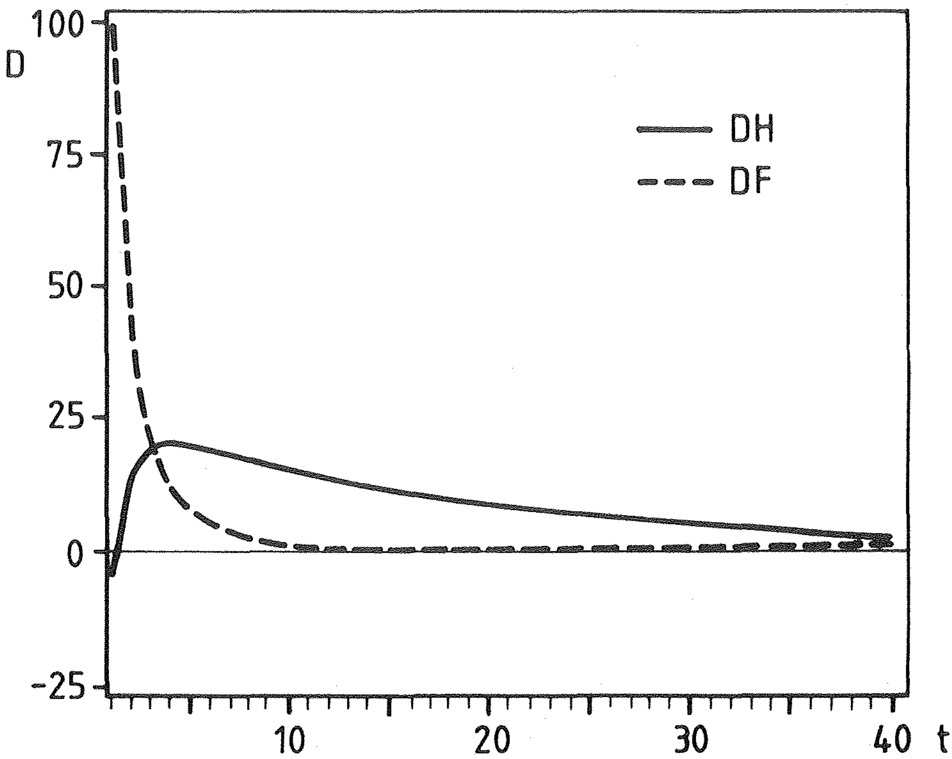
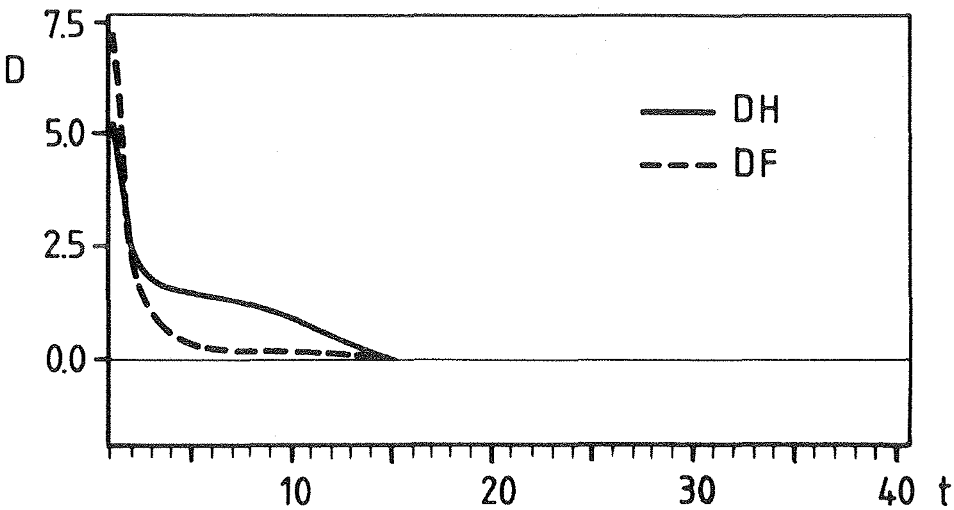


Figure 2: The densities for households and firms estimated for Viborg. (Each interval indicates 250 m; axis begins 125 m from the center.)



AN OVERALL MODEL, INCLUDING THE DENSITY FUNCTIONS OF ALL TOWNS, USING THE EXPANSION METHOD

The estimated coefficients of the above-mentioned density functions for 22 Danish towns can be seen as functions of the population size in general.

$$a_j = f_j\,(POP) \qquad (3)$$

where a_j is coefficient number j in (2).

If it is correct that the density crater, as this investigation suggests, will first open when the town under specific conditions reaches a certain size, then a_0 in (2) might be assumed to begin at a given positive value which rises with a rising population to a certain level, then declines, and finally becomes negative. The coefficient a_1 must be negative for small towns, and with a rising population, should decline numerically and change sign when the density crater opens. The description of the crater status is completed by a_2 and possibly a_3.

These considerations lead to a model where the development in the coefficients of the individual town by increasing population follows an equation of the type

$$a_j = \beta_{0j} + \beta_{1j}POP + \beta_{2j}POP^2 \qquad (4)$$

for $j = 0, 1, 2$ and 3.

Income must be supposed to influence the density profile as well. Income was included in this project as the average income in the town, and is related to the occupational structure in the town. At the 1986 income level, the share of manufacturing occupations was rising, going from low income towns to medium income towns, and declining from medium income to high income. The counterpart to manufacturing is service, administration and hotels, the share of which is declining slightly from low income to medium income, and rapidly increasing going from medium income to high income. In addition to the indirect effect of the income level on density through the industrial structure, there must be direct effects where income influences the demand of the citizens for space and proximity to the center. An attempt is therefore made to catch the income effect on density by a second degree polynomium of income. "Trade and hotel business" was the group most influenced by the income level, with the second most influenced, "manufacturing." Since the two groups complement each other, only "trade and hotel business" was included as an indicator for the industrial structure. Because the income effect might vary with city size, the coefficient equation was formed as:

$$a_j = b_{0j} + \beta_{1j}lP + b_{3j}y + b_{4j}y^2 + B_{5j}ylP \qquad (5)$$

or

$$a_j = b_{0j} + B_{1j}lP^2 + b_{3j}y + b_{4j}sah \qquad (6)$$

where j = 0,1,2,3
y – income
sha – share of urban day-time population occupied in trade and hotel business
lP – log(POP).

The expanded model is now given the form inserting (5) in (2):

$$\begin{aligned} D = {} & (b_{00} + b_{10}lP + b_{20}lP^2 + b_{30}y + b_{40}y^2 + b_{50}ylP) \\ & (b_{01} + b_{11}lP + b_{21}lP^2 + b_{31}y + b_{41}y^2 + b_{51}ylP)lt \\ & (b_{02} + b_{12}lP + b_{22}lP^2 + b_{32}y + b_{42}y^2 + b_{52}ylP)lt^2 \\ & (b_{03} + b_{13}lP + b_{23}lP^2 + b_{33}y + b_{43}y^2 + b_{53}ylP)lt^3 \end{aligned} \quad (7)$$

An alternative use of the indirect effect:

$$sha = bb_0 + bb_1y + bb_2y^2 \quad (8a)$$

$$\begin{aligned} D = {} & (b_{00} + b_{10}lP + b_{20}lP^2 + b_{30}y + b_{40}sha + b_{50}ylP) \\ & (b_{01} + b_{11}lP + b_{21}lP^2 + b_{31}y + b_{41}sha + b_{51}ylP)lt \\ & (b_{02} + b_{12}lP + b_{22}lP^2 + b_{32}y + b_{42}sha + b_{52}ylP)lt^2 \\ & (b_{03} + b_{13}lP + b_{23}lP^2 + b_{33}y + b_{43}sha + b_{53}ylP)lt^3 \end{aligned} \quad (8b)$$

The results of the overall estimation with household density gave the equation:

$$\begin{aligned} DH = {} & (\underset{(1.74)}{2312.04} - \underset{(-2.43)}{63.28lP} - \underset{(-5.37)}{8.28lP^2} - \underset{(-1.62)}{36.21y} + \underset{(1.66)}{.1585y^2} + \underset{(2.11)}{.056ylP}) \\ & (\underset{(-1.71)}{-710.99} + \underset{(5.27)}{55.64lP} + \underset{(1.49)}{9.91y} - \underset{(-1.57)}{.0449y^2})lt \\ & (\underset{(1.77)}{63.61} - \underset{(-5.98)}{11.58lP} + \underset{(4.80)}{.45lP^2} - \underset{(-1.37)}{.67y} + \underset{(1.44)}{.0033y^2})lt^2 \\ & (\underset{(-1.45)}{-1.32} + \underset{(5.46)}{.67lP} - \underset{(-4.39)}{.039lP^2} - \underset{(-.39)}{2.7030y^2})lt^3 \end{aligned}$$

$R^2 = .674 \quad \bar{R}^2 = .661$

The results of the overall estimation with firm density gave the equation:

$$sha = \underset{(7.79)}{1.3080} - \underset{(-6.93)}{.01998y} + \underset{(7.14)}{.00000882y^2}$$

$R^2 = .164 \quad \bar{R} = .160$

$$
\begin{aligned}
DF = \; & (23.66 - 527.991P + 74.361P^2 + 1.53y + 4580.97sha + .0651y1P) \\
& (2.67) \quad (-5.30) \quad (7.76) \quad (2.59) \quad (3.50) \quad (2.58) \\
& (192.08 - 26.751P - 4.21P^2 - 1893.32sha)1t \\
& (4.30) \quad (-6.28) \quad (-2.52) \quad (-3.16) \\
& (-23.681P + 3.201P^2 + .025y + 259.13sha)1t^2 \\
& (3.60) \quad (5.17) \quad (2.05) \quad (2.87) \\
& (.971P - .121P^2 - 11.75sha)1t^3 \\
& (3.07) \quad (-4.33) \quad (-2.63)
\end{aligned}
$$

$R^2 = .842 \quad \bar{R}^2 = .836$

THE EMPIRICALLY ESTIMATED COEFFICIENT FUNCTIONS

Referring to equation (1), it is obvious that the values of a_0 and a_1 must follow the patterns below, depending on whether the density crater has opened or not:

	density crater	
coefficient	not open	open
a_0	> 0	low or < 0
a_1	< 0	> 0

The dependence of a_0, a_1, a_2 and a_3 on POP and y is shown graphically in Figures 3 and 4.

The effect of income is small on the density pattern of households, but great on the density pattern of firms. The reason may be that the direct and the indirect effects to some degree outweigh each other for households, while for firms the indirect effect of income through changed industrial structure is dominant. This suggestion enjoys some support from a study of Tkocz and Kristensen (1990).

THE DENSITY PROFILES AS POPULATION INCREASES

Of special interest in this study is the interaction between the density of firms and the opening of the household density crater. As indicated to some degree above with the examples of Viborg and Copenhagen, the value of a_0 is always positive and a_1 is always negative for firms.

The total profile as the town increases can now be drawn up. Different income levels give different scenarios of the development. The pattern of towns where some, the big towns, have no crater, while others, very small towns, have, gives the impression that the opening of the crater might not be gradual, but will happen as an "explosion" when released. In Figure 5 and 6, a gradual opening and a constant income level of 120 is supposed.

Figure 3: The coefficient functions for household densities as functions of population and income level.

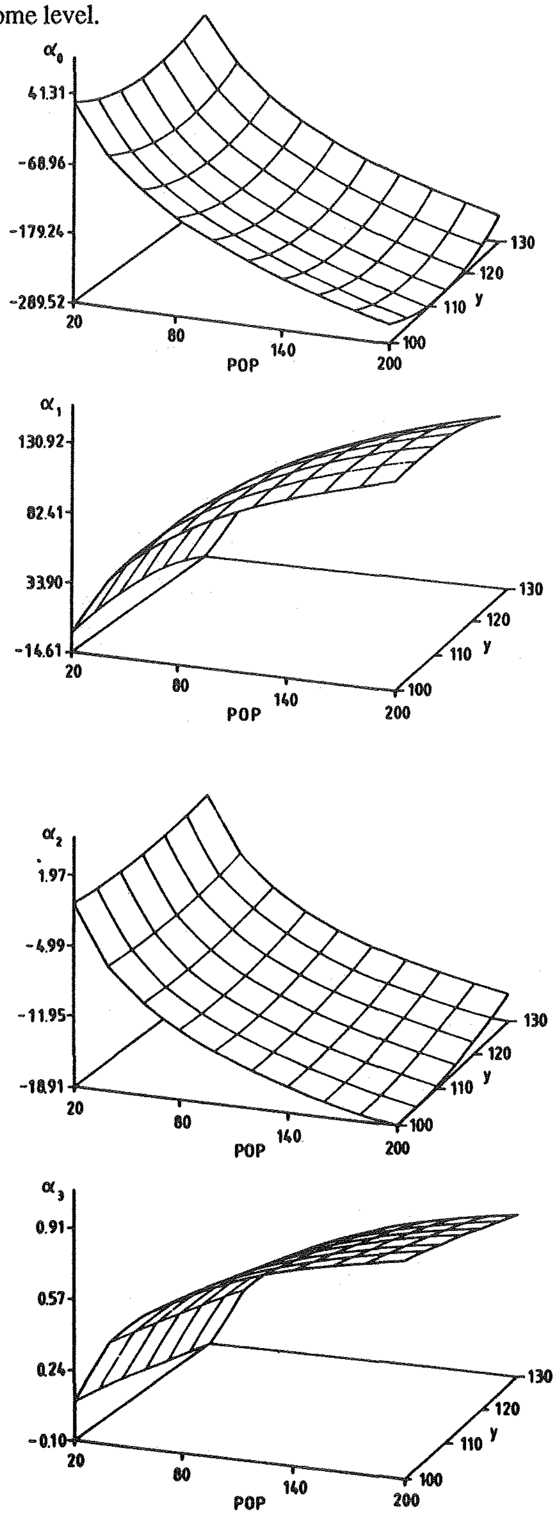


Figure 4: The coefficient functions for firm densities as functions of population and income level.

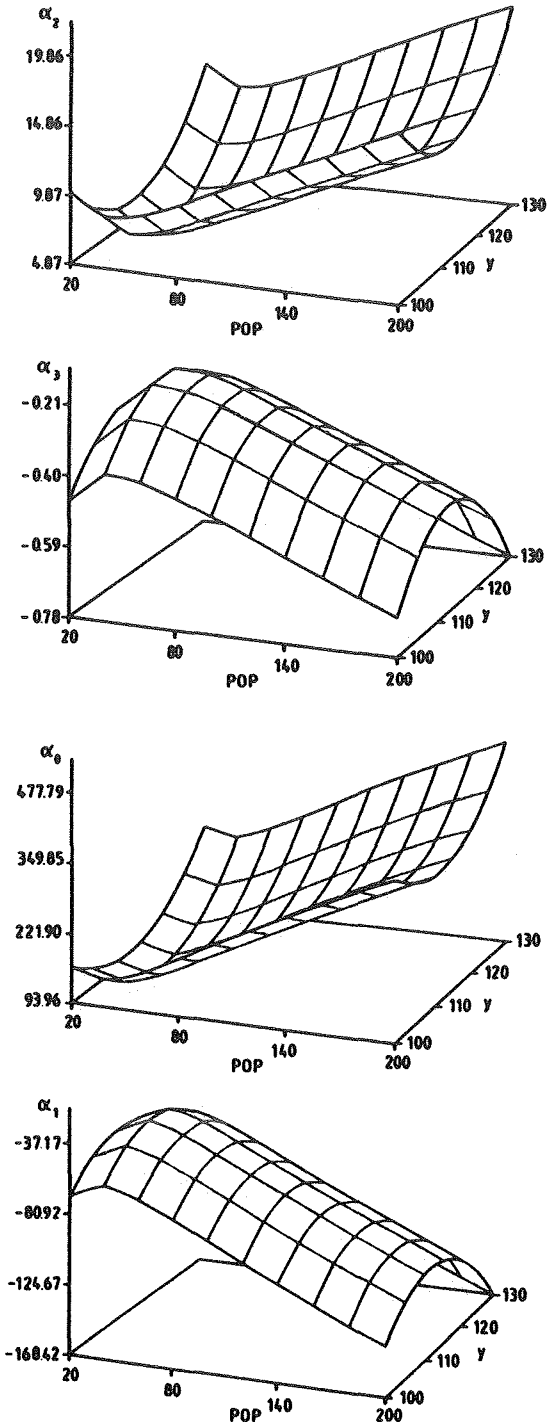


Table 1 shows the name of the included cities, the population size, and average income. From the estimations of density profiles from the individual towns, we know whether the population density crater has opened or not.

Table 1: The opening of population density crater seen in relation to town size and town average income.

Name of town	Population	Average income	Crater opened
Copenhagen	1,170,116	125,300	yes
Århus	195,152	114,500	yes
Odense	137,286	108,000	no
Ålborg	113,650	109,800	yes
Esbjerg	71,112	116,100	yes
Randers	55,563	108,000	yes
Horsens	46,735	111,100	yes
Vejle	44,253	115,700	yes
Helsingør	43,696	125,600	no
Kolding	43,692	117,000	no
Roskilde	38,606	127,900	yes
Næstved	38,159	113,200	yes
Fredericia	35,059	114,800	no
Silkeborg	34,172	116,600	yes
Viborg	29,409	113,600	no
Slagelse	28,624	111,000	no
Herning	28,618	117,900	yes
Svendborg	25,243	105,700	no
Hjørring	23,813	111,500	yes
Nakskov	15,909	100,000	no
Nyborg	15,296	107,400	no
Frederikssund	13,774	130,800	yes

The possibility of a disharmonic crater opening process, where the probability for opening increases with increasing population, but where the opening itself is unexplained by the model, makes it worthwhile trying an alternative formulation of (7). In the alternative formulation, the opening is caught up by a dummy, based, as shown above, on the individual estimated equations. The best estimates were obtained by making the constants in the expanded coefficients dummies which take the value, 0, when the crater has opened, and 1, when it has not.

Figure 5: Density profiles for households as population rises at constant income level.

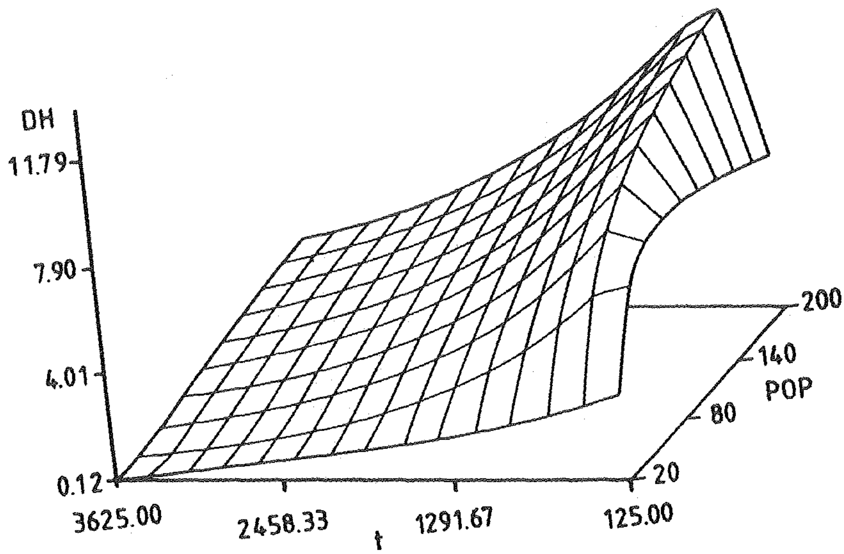
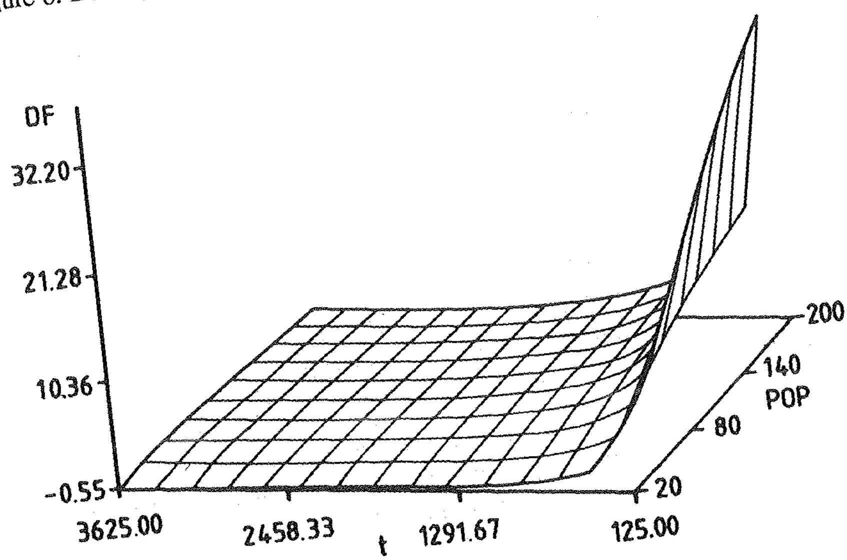


Figure 6: Density profile for firms as population rises at constant income level.



The estimated relationship for household density and distance to center then becomes:

$$
\begin{array}{llllllll}
DH = & (257.68DC & - 45.59lP & - 7.60lP^2 & - 4.26y & + .043y^2 & + .040ylP) \\
 & (4.32) & (-2.01) & (-6.84) & (-2.66) & (2.78) & (2.78) \\
 & (-109.61DC & + 46.23lP & & + 1.07y & - .016y^2)lt & \\
 & (-4.05) & (4.51) & & (2.24) & (-2.67) & \\
 & (15.34DC & - 9.92lP & + .41lP^2 & - .063y & + .0019y^2)lt^2 & \\
 & (3.82) & (-5.45) & (6.38) & (-1.80) & (2.58) & \\
 & (-.70DC & + .58lP & - .035lP^2 & & - .000083y^2)lt^3 & \\
 & (-3.61) & (5.38) & (-5.86) & & (-2.52) & \\
 & R^2 = .695 & \bar{R}^2 = .682 & & & &
\end{array}
$$

where DC is a dummy for crater opening.

The estimated relationship has a higher significance level than the "combined" estimation above. This estimation, however, demands a priori knowledge as to whether there has been an opening. The estimated equations leave the impression that the town density pattern follows a development, as indicated by making DC=1, until a certain point, which differs from town to town; then the crater opens and a new line is followed.

CONCLUSION

This article shows that it is fruitful for understanding the urban economic structure to combine population density with density of business firms. Although it is not possible to model this interrelationship on the material treated, the comparative description in the two densities obviously gives a more complete picture of the opening of the density crater. The inclusion of income as an explanatory variable opens a complicated pattern of direct effects, where rising income levels change the evaluation of proximity to the center and demand for space, and an indirect effect, by a changed occupational pattern. There is, however, a great degree of multicolinearity between income and industrial structure which makes a separation of the two effects difficult on the basis of the considered data. It must be supposed that a cross-section analysis such as this has its advantages in describing the indirect effect.

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